Title: On irreducible maps and extremally disconnected spaces

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PRACE NAUKOWE UNIWERSYTETU SLĄSKIEGO W KATOWICACH NR 30
PRACE MATEMATYCZNE III, 1973

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ON IRREDUCIBLE MAPS AND EXTREMALLY DISCONNECTED SPACES

The aim of this paper is to discuss various kinds of irreducible maps, to give a characterization of extremally disconnected spaces with the aid of these maps, and to prove some new facts on the Iliadis extremally disconnected resolution. The paper ends with a general definition of an extremally disconnected resolution of a Hausdorff space including the Iliadis resolution [2] modified by Mioduszewski and Rudolf [4].

§ 1. COMPLETELY IRREDUCIBLE MAPS. Let (X, τ) be a given topological space. An open subset A of X is said to be *regularly open* if $A = Int_{\tau} Cl_{\tau} A$. A closed set is said to be *regularly closed* if its complement is a regularly open set. It is easy to see that a closed set is regularly closed if it is the closure of an open set.

A map $f: X \xrightarrow{\text{onto}} Y$ is said to be *irreducible* ([4] page 26) if for each regularly closed subset A of X

$$A \neq X \text{ implies } \mathrm{Cl} f(A) \neq Y. \tag{1}$$

A map $f: X \xrightarrow{\text{onto}} Y$ will be said to be *completely irreducible* if for each closed (not necessarily regularly closed) subset A of X

$$A \neq X \text{ implies } \mathrm{Cl} f(A) \neq Y. \tag{2}$$

Clearly, every completely irreducible map is irreducible. The converse implication is not true.

Example. Let τ be the natural topology on $I = (0,1)$. Let τ' be the topology on $(0,1)$ generated by the family $\tau \cup \{W\}$, where W is the set of all rational numbers of I. The contraction from τ' to τ i.e. the map $(I, \tau') \xrightarrow{\text{onto}} (I, \tau)$ being the identity on the set I is not completely irreducible because the set of all irrational numbers is closed in τ' and dense in τ. On the other hand, each regularly closed set in τ' is regularly closed in τ. Hence there does not exist a regularly closed set in τ' which is different from I and which is dense in τ. Therefore the contraction from τ' to τ is irreducible.

Clearly, a map $f: X \xrightarrow{\text{onto}} Y$ is completely irreducible iff there exists a base β of open sets such that for each non-empty set $U \in \beta$ we have $\mathrm{Clf}\,(X \setminus U) \neq Y$.

A Hausdorff space X is said to be *semi-regular* if the family of all regularly open sets forms a base in X. Every regular space is semi-regular. Hence if a space X is semi-regular, then every irreducible map $f: X \xrightarrow{\text{onto}} Y$ is completely irreducible.

A map $f: X \xrightarrow{\text{onto}} Y$ is said to be r.o.-minimal [4] if the topology in X is generated by the family $\{f^{-1}(U) \cap V : U$ is open in Y and V is regularly open in $X\}$.

THEOREM 1. *If a map $f: X \xrightarrow{\text{onto}} Y$ is irreducible and r.o.-minimal, then f is completely irreducible.*

Note: Although the theorem follows from some theorems from [4], we give here a direct proof.

Proof. If U is open in Y and V is regularly open in X, then the set $W = f^{-1}(U) \cap V$ is a base open set in X. Suppose that $\mathrm{Clf}\,(X \setminus W) = Y$. It suffices to show that $W = \emptyset$.

We have

$$Y = \mathrm{Clf}\,(X \setminus (f^{-1}(U) \cap V)) = \mathrm{Clf}\,[(X \setminus f^{-1}(U)) \cup (X \setminus V)] =$$
$$\mathrm{Clf}\,[f^{-1}(Y \setminus U)] \cup \mathrm{Clf}\,(X \setminus V) = (Y \setminus U) \cup \mathrm{Clf}\,(X \setminus V).$$

Then

$$U \cap (Y \setminus \mathrm{Clf}\,(X \setminus V)) = \emptyset.$$

Hence

(3) $$f^{-1}(U) \cap \mathrm{Clf}^{-1}(X \setminus \mathrm{Clf}\,(X \setminus V)) = \emptyset.$$

Consider the set $\mathrm{Clf}^{-1}(Y \setminus \mathrm{Clf}\,(X \setminus V)) \cup (X \setminus V)$. It is a regularly closed set and we have

$$\mathrm{Clf}\,[\mathrm{Clf}^{-1}(Y \setminus \mathrm{Clf}\,(X \setminus V)) \cup (X \setminus V)] = \mathrm{Clf}\,[\mathrm{Clf}^{-1}(Y \setminus \mathrm{Clf}\,(X \setminus V))] \cup$$
$$\cup\, \mathrm{Clf}\,(X \setminus V) \supset [Y \setminus \mathrm{Clf}\,(X \setminus V)] \cup \mathrm{Clf}\,(X \setminus V) = Y.$$

Since the map f is irreducible, we get

$$\mathrm{Clf}^{-1}(Y \setminus \mathrm{Clf}\,(X \setminus V)) \cup (X \setminus V) = X.$$

Hence

$$V \subset \mathrm{Clf}^{-1}(Y \setminus \mathrm{Clf}\,(X \setminus V)),$$

and from (3) we get

$$W = f^{-1}(U) \cap V = \emptyset.$$

E x a m p l e. There exist completely irreducible maps which are not r.o.-minimal. Let us consider the space $X_0 = \left(\{0\} \cup \left\{\frac{1}{n}\right\}_{n=1}^{\infty}\right) \times \left\{\frac{1}{n}\right\}_{n=1}^{\infty}$. Let $X = X_0 \cup \{a\}$ where $a \notin X_0$. Topology in X_0 is the usual product topology and baisc neighbourhoods of the point a are the sets $U_k = \{a\} \cup \left(\left\{\frac{1}{n}\right\}_{n=1}^{\infty} \times \left\{\frac{1}{n}\right\}_{n=k}^{\infty}\right)$. This topology will be denoted by τ. The space (X, τ) is a Hausdorff but not semi-regular space. Let τ' be the topology on X generated by the family of all regularly open subsets of X. It is easy to see that the contraction $i_X : (X, \tau) \xrightarrow{\text{onto}} (X, \tau')$ is completely irreducible but not r.o.-minimal.

A space is said to be *extremally disconnected* (Stone [5]) if the closure of each of its open subsets is open. A Hausdorff space X is extremally disconnected iff for every two disjoint sets U, V which are open in X we have the equality $\mathrm{Cl}U \cap \mathrm{Cl}V = \emptyset$

J. Flachsmeyer studied in [1] irreducible maps in the following (common) sense :

(4) a map $f : X \xrightarrow{\text{onto}} Y$ is said to be *irreducible* if, for each closed subset A of X,

$$A \neq X \text{ implies } f(A) \neq Y.$$

Recall that a map is said to be *perfect* if it is closed and if counter-images of points are compact. Hence if an irreducible map (in the common sense) is perfect, then it is completely irreducible (the converse is, of course, not true).

In [1] it was proved that every perfect irreducible (in the common sense) map onto an extremally disconnected Hausdorff space is a homeomorphism. We shall show more, namely.

THEOREM 2. *If X is a Hausdorff space and Y is an extremally disconnected Hausdorff space, then every completely irreducible map $f : X \xrightarrow{\text{onto}} Y$ is one-to-one (and hence it is a contraction).*

P r o o f. Suppose, on the contrary, that there exist two various points x and x' in X such that $f(x) = y = f(x')$. Since X is Hausdorff, there exist two open sets U and U' such that $x \in U$, $x' \in U'$ and $U \cap U' = \emptyset$.

Consider the open set $V = Y \setminus \mathrm{Cl}f(X \setminus U)$. We shall show that $y \in \mathrm{Cl}V$. In fact, the set $\mathrm{Cl}f^{-1}(V) \cup (X \setminus U)$ is closed in X.
We have

$$\mathrm{Cl}f\,[\mathrm{Cl}f^{-1}(V) \cup (X \setminus U)] = \mathrm{Cl}f\,[\mathrm{Cl}f^{-1}(V)] \cup \mathrm{Cl}f\,(X \setminus U) =$$
$$\mathrm{Cl}f\,(\mathrm{Cl}f^{-1}(V)) \cup (Y \setminus V) \supset V \cup (Y \setminus V) = Y.$$

From the assumption that f is completely irreducible it follows that

$$\mathrm{Cl}f^{-1}(V) \cup (X \setminus U) = X.$$

Then

$$U \subset \mathrm{Cl} f^{-1}(V).$$

Hence

$$y \in f(U) \subset f(\mathrm{Cl} f^{-1}(V)) \subset \mathrm{Cl} f(f^{-1}(V)) = \mathrm{Cl} V.$$

In the same way $y \in \mathrm{Cl} V'$, where $V' = Y \setminus \mathrm{Cl} f(X \setminus U')$ by definition. On the other hand

$$V \cap V' = [Y \setminus \mathrm{Cl} f(X \setminus U)] \cap [Y \setminus \mathrm{Cl} f(X \setminus U')] = Y \setminus [\mathrm{Cl} f(X \setminus U) \cup \mathrm{Cl} f(X \setminus U')] = Y \setminus \mathrm{Cl} f[X \setminus (U \cap U')] = Y \setminus \mathrm{Cl} f(X) = \emptyset.$$

Since Y is extremally disconnected, we have

$$\mathrm{Cl} V \cap \mathrm{Cl} V' = \emptyset.$$

But $y \in \mathrm{Cl} V \cap \mathrm{Cl} V'$, contradiction.

§ 2. CHARACTERIZATION OF EXTREMALLY DISCONNECTED SPACES. Iliadis in [2] constructed for every Hausdorff space so called extremally disconnected resolution (absolute). Let us shortly recall this construction.

Let X be a Hausdorff space. Denote by ωX the set of all convergent open ultrafilters (i.e., ultrafilters containing all neighbourhoods of some point of X). The family of sets $O_U = \{\xi \in \omega X : U \in \xi\}$, where U is open in X, is a base of topology in ωX. There is a natural map $\omega^X : \omega X \xrightarrow{\text{onto}} X$ defined by the formula

$$\omega^X(\xi) = \bigcap \{\mathrm{Cl} U : U \in \xi\}. \tag{5}$$

The space ωX proves to be an exstremally disconnected Hausdorff space, and the map ω^X irreducible but only Θ-continuous (in the sense of Fomin; cf [3]). In paper [4] the Iliadis construction was modified. The modified Iliadis resolution αX is the set ωX with the topology generated by the sets $O_V \cap (\omega^X)^{-1}(U)$. α^X, is the map equal to ω^X in set-theoretical sense, carrying the space αX onto X. It was proved there that αX is an extremally disconnected Hausdorff space and the map α^X is continuous and irreducible.

LEMMA 1. *The map α^X is an r.o.-minimal irreducible map (hence a completely irreducible map).*

P r o o f. From the definition of O_U it follows immediately that for each open $U \subset X$ the set O_U is closed-open in αX (see [3] page 50). Hence the map α^X is r.o.-minimal. Then in virtue of Theorem 1 α^X is a completely irreducible map.

A map $f : X \xrightarrow{\text{onto}} Y$ is said to be *skeletal* ([4], page 13) if for each open subset V of X we have

$$\mathrm{Int} f^{-1}(\mathrm{Cl} V) \subset \mathrm{Cl} f^{-1}(V). \tag{6}$$

In [4] it was proved that every irreducible map is skeletal and that for any skeletal map $f: X \xrightarrow{\text{onto}} Y$ there exists a unique map $af: aX \xrightarrow{\text{onto}} aY$ such that the diagram

$$
(7) \qquad \begin{array}{ccc} aX & \xrightarrow{af} & aY \\ {\scriptstyle a^X}\downarrow & & \downarrow{\scriptstyle a^Y} \\ X & \xrightarrow{f} & Y \end{array}
$$

commutes, i.e. $f \circ a^X = a^Y \circ af$.

Topologies on a given set having the same family of regularly open subsets are said to be *r.o.-equivalent.*

A contraction from τ to τ' is said to be *conservative* if

(8) for each U' open in τ' we have $\mathrm{Cl}_{\tau'} U' = \mathrm{Cl}_{\tau} U'$.

It was proved in [4] that topologies τ and τ' are r.o.-equivalent iff

(9) for each U open in τ we have $\mathrm{Cl}_{\tau'} U = \mathrm{Cl}_{\tau} U$.

LEMMA 2. *For any continuous maps $f: X \xrightarrow{\text{onto}} Y$ and $g: Y \xrightarrow{\text{onto}} Z$, $h = g \circ f$ is completely irreducible iff f and g are completely irreducible.*

P r o o f. 1. Let us asumme that $f: X \xrightarrow{\text{onto}}$ and $g: Y \xrightarrow{\text{onto}} Z$ are completely irreducible maps. Let F be a closed subset of X and $\mathrm{Cl}h(F) = Z$. We have $\mathrm{Cl}(g(f(F))) = Z$, hence $\mathrm{Cl}g(\mathrm{Cl}f(F)) = Z$. By hypothesis the maps g is completely irreducible, whence $\mathrm{Cl}f(F) = Y$ and in virtue of the complete irreducibility of the map f we get $F = X$. Thus the map $h = g \circ f$ *is* completely irreducible.

2. Let h be a completely irreducible map. We shall show that f and g are completely irreducible.
In fact, let F be a closel subset of X and $\mathrm{Cl}f(F) = Y$. We have

$$Z = g(Y) = g(\mathrm{Cl}f(F)) \subset \mathrm{Cl}(g(f(F)) = \mathrm{Cl}h(F).$$

The map h is completely irreducible, $F = X$. Thus f is completely irreducible.

Let H be a closed subset of Y and $\mathrm{Cl}g(H) = Z$. Put $F = f^{-1}(H)$, a closed subset of X. We have $f(F) = H$. Hence

$$\mathrm{Cl}g(f(F)) = \mathrm{Cl}g(H) = Z.$$

Hence $F = X$ which implies that $H = Y$. Thus g is completely irreducible.

LEMMA 3. *If X is a Hausdorff space and Y is an extremally disconnected space, then every completely irreducible map $f: X \xrightarrow{\text{onto}} Y$ is a conservative contraction.*

P r o o f. In virtue of Theorem 2, the map f is a contraction. The map f being irreducible is a skeletal map. Then for each V open in Y we have $\mathrm{Int} f^{-1}(\mathrm{Cl} V) \subset \mathrm{Cl} f^{-1}(V)$. Since the space Y is extremally disconnected, $\mathrm{Cl} V$ is open in Y. Thus $f^{-1}(\mathrm{Cl} V) \subset \mathrm{Cl} f^{-1}(V)$. The converse inclusion holds for every continuous map. Thus we have $f^{-1}(\mathrm{Cl} V) = \mathrm{Cl} f^{-1}(V)$, which means that f is conservative.

LEMMA 4. *If a contraction* $i_X : (X, \tau) \xrightarrow{\text{onto}} (X, \tau')$ *is irreducible and the spaces* (X, τ) *and* (X, τ') *are extremally disconnected, then the topologies* τ *and* τ' *are r.o.-equivalent.*

P r o o f. Since the contraction from τ to τ' is irreducible, it is a skeletal map. Thus for each $U' \in \tau'$ we have

$$\mathrm{Int}_\tau \mathrm{Cl}_{\tau'} U' \subset \mathrm{Cl}_\tau U'.$$

Since (X, τ') is an extremally disconnected space, $\mathrm{Cl}_\tau U'$ is open in (X, τ') and we have

$$\mathrm{Cl}_{\tau'} U' \subset \mathrm{Cl}_\tau U'.$$

In virtue of the inclusion $\tau' \subset \tau$ we have

$$\mathrm{Cl}_{\tau'} U' \supset \mathrm{Cl}_\tau U'.$$

Hence the contraction from τ to τ' is conservative. From (8) it follows that each regulary closed set in τ' is regularly closed in τ.

Let $U \in \tau$ be a non-empty open subset of X. Since (X, τ) is extremally disconnected, U is closed-open. In virtue of the irreducibility the set $X \setminus U$ is not dense in (X, τ') and we have

$$\mathrm{Int}_{\tau'} U \neq \emptyset.$$

From (8) it follows that

$$U = \mathrm{Cl}_\tau \mathrm{Int}_\tau U \supset \mathrm{Cl}_\tau \mathrm{Int}_{\tau'} U = \mathrm{Cl}_{\tau'} \mathrm{Int}_{\tau'} U.$$

Suppose that the difference $U \setminus \mathrm{Cl}_{\tau'} \mathrm{Int}_{\tau'} U = A$ is non-empty. Since (X, τ') is extremally disconnected, $\mathrm{Cl}_{\tau'} \mathrm{Int}_{\tau'} U$ is closed-open in τ'. Thus A is a closed-open subset of (X, τ). In virtue of the irreducibility we have

$$\mathrm{Int}_{\tau'} A = \emptyset.$$

Thus

$$\mathrm{Int}_{\tau'} A = \mathrm{Int}_{\tau'} U \cap \mathrm{Int}_{\tau'} (X \setminus \mathrm{Cl}_{\tau'} \mathrm{Int}_{\tau'} U) \subset$$

$$\mathrm{Int}_{\tau'} U \cap (X \setminus \mathrm{Int}_{\tau'} U) = \emptyset.$$

which is a contradiction.

Thus each regularly open set in τ is regularly open in τ'.

THEOREM 3. *If a map* $f : X \xrightarrow{\text{onto}} Y$ *is completely irreducible, then the topologies in* αX *and* $\alpha f(\alpha X)$ *are r.o.-equivalent.*

Proof. The map $f: X \xrightarrow{\text{onto}} Y$ is skeletal, thus the diagram (7) may be completed. The map α^X is completelly irreducible. In virtue of Lemma 2, the map $fo\alpha^X$ is completly irreducible, hence $\alpha^X o\alpha f$ is completely irreducible. Then from Lemma 2 it follows that αf is completely irreducible. In virtue of Lemma 4 the topologies in αX and $\alpha f(\alpha X)$ are r.o.-equivalent.

The following theorem gives a certain description of the extremally disconnected spaces.

THEOREM 4. *A Hausdorff space Y is extremally disconnected iff each completely irreducible map onto Y is a conservative contraction.*

Proof. One of the implications coincides with Lemma 3. Suppose that Y is a Hausdorff space and that each completely irreducible map onto Y is a conservative contraction. The map $\alpha^X : \alpha Y \xrightarrow{\text{onto}} \alpha Y$ is completely irreducible, hence it is a conservative contraction. Let U and V be open subsets of Y such that $U \cap V = \emptyset$. Then we hawe $\mathrm{Cl}_{\alpha Y} U \cap \mathrm{Cl}_{\alpha Y} V = \emptyset$. But we have $\mathrm{Cl}_{\alpha Y} U = \mathrm{Cl}_{\alpha Y} U$ and $Cl_{\alpha Y} V = \mathrm{Cl}_{\alpha Y} V$. Thus $\mathrm{Cl}_Y U \cap \mathrm{Cl}_Y V = \emptyset$ and this means that Y is extremally disconnected.

§ 3. REGULAR EXTREMALLY DISCONNECTED SPACES. THE SET OF EXETREMALLY DISCONNECTED RESOLUTIONS OF A GIVEN HAUSDORFF SPACE. Let us note that for semi-regular (in particular, for regular) spaces each irreducible map is completely irreducible and each conservative contraction is a homeomorphism, since the family of all regularly open subsets forms a base in such a space.

LEMMA 5. *If a space X is regular, then the resolution ωX coincides with αX.*

Proof. Suppose that $\xi \in (\alpha^X)^{-1}(V)$, where V is open in X. Let $y = \alpha^X(\xi) \in V$. In virtue of the regularity of the space X, there exists an open sét U such that $y \in U \subset \mathrm{Cl}U \subset V$. Since $y \in U$ we have $U \in \xi$, and hence $\xi \in O_U$. It is easy to se that $\alpha^X(O_U) = \mathrm{Cl}U$. Then we have $\alpha^X(O_U) \subset V$; hence $\xi \in O_U \subset (\alpha^X)^{-1}(V)$. This means that the sets O_U form a base in αX. Consequently the topologies in αX and in ωX are the same.

THEOREM 5. *If a space X is regular, then the (modifield) Iliadis resolution αX is also regular.*

Proof. Let X be a regular space. In virtue of Lemma 5 the resolution αX coincides with ωX. In [3] (Theorem 10, page 64) it was proved that ωX is regular.

In virtue of Theorem 3 and Theorem 5 we have

COROLLARY. If a map $f: X \xrightarrow{\text{onto}} Y$ is irreducible and X is a regular space, then the map $\alpha f : \alpha X \longrightarrow \alpha Y$ is an embedding.

In [3] similar fact was proved under the additional hypothesis that f is perfect.

The Iliadis extremally disconnected resolution is given by an individual construction. Generally, by an *extremally disconnected resolution* of a given Hausdorff space X we mean any r.o.-minimal irreducible map $\mu^X : \mu X \xrightarrow{\text{onto}} X$ such that μX is an extremally disconnected Hausdorff space.

From Lemma 1 it follows that the Iliadis resolution αX is extremally disconnected in the sense of our definition.

THEOREM 6. *If a space X is extremally disconnected, then for every exstremally disconnected resolution μX the map μ^X is a homeomorphism.*

P r o o f. Let $\mu^X : \mu X \xrightarrow{\text{onto}} X$ be an extremally disconnected resolution of a space X. From Theorem 1 it follows that the map μ^X is completely irreducible, hence in virtue of Theorem 2 μ^X is a contraction. Since the spaces X and μX are extremally disconnected, the topologies in X and in μX are r.o.-equivalent in virtue of Lemma 5. The map μ^X is r.o.-minimal, thus μ^X must be a homeomorphism.

THEOREM 7. *For every extremally disconnected resolution μX there exists a unique map $\varphi : \mu X \to \alpha X$ which is a conservative contraction onto $\varphi(\mu X)$ and fills up the diagram*

(10)
$$\begin{tikzcd} \alpha X \arrow[d, "\alpha^X"'] & \\ X & \mu^X \arrow[l, "\mu^X"] \arrow[ul, dashed, "\varphi"'] \end{tikzcd}$$

In addition the topologies in $\varphi(\mu X)$ and μX are r.o.-equivalent.

P r o o f. From [4] (3.4. page 33) it follows that there exists a unique map $\varphi : \mu X \to \alpha X$ filling up diagram (10). In virtue of Lemma 2, φ is a completely irreducible map. Since αX is extremally disconnected space, φ is a conservative contraction (Theorem 4). In virtue of Lemma 4, topologies in μX and $\varphi(\mu X)$ are r.o.-equivalent.

This theorem easily implies that if an extremally disconnected resolution μX is a compact space, then μX coincides with αX.

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O ODWZOROWANIACH NIEPRZYWIEDLNYCH I PRZESTRZENIACH EKSTREMALNIE NIESPÓJNYCH

Streszczenie

Przedmiotem pracy są przestrzenie ekstremalnie niespójne i pewne rodzaje odwzorowań nieprzywiedlnych, m. in. odwzorowania nieprzywiedlne w sensie Mioduszewskiego i Rudolfa. Praca zawiera zewnętrzną charakteryzację przestrzeni ekstremalnie niespójnych przy pomocy tych odwzorowań. Podana jest ogólna definicja nakrycia ekstremalnie niespójnego dowolnej przestrzeni Hausdorffa X. Jednym z takich nakryć jest $\alpha^X : \alpha X \xrightarrow{\text{na}} X$ zdefiniowane przez Iliadisa w [2] i zmodyfikowane przez Mioduszewskiego i Rudolfa w [4], gdzie udowodniono, że (zmodyfikowane) nakrycie Iliadisa jest funktorem, który każdemu odwzorowaniu szkieletowemu $f : X \xrightarrow{\text{na}} Y$ przyporządkowuje odwzorowanie $\alpha f : \alpha X \xrightarrow{\text{na}} \alpha Y$. Iliadis i Fomin w [3] udowodnili, że jeśli f jest odwzorowaniem nieprzywiedlnym i doskonałym, to αf jest homeomorfizmem. W tej pracy pokazuję, że jeśli $f : X \xrightarrow{\text{na}} Y$ jest odwzorowaniem nieprzywiedlnym, to topologie w αX i $\alpha f(\alpha X)$ są r.o.-równoważne oraz że αf jest zanurzeniem jeśli tylko X jest przestrzenią regularną.

Oddano do Redakcji 26. 3. 1970 r.